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The Marxian equations of reproduction and modern economics*

I

Before we start dealing with the proper subject of this paper we shall modify somewhat the Marxian division of economy into departments in order to simplify our argument and in order to focus on the basic problem of the reproduction schemes.

First, instead of including producer goods in Department 1, we will assume that it covers the total value of gross investment inclusive of the respective raw materials. Thus this department represents the integrated production of all final non-consumer products. (We disregard in our argument as does Marx — when he deals with reproduction schemes — both foreign trade and government revenue and expenditure.)

Second, we treat likewise the consumer goods, *i.e.*, we include in the department which covers their output the production of respective raw materials from top to bottom. Moreover, fully in the Marxian spirit, we distinguish the following two departments : Department 2 producing consumer goods for capitalists and Department 3 producing wage goods.

We obtain thus the following " tableau économique " of the national income where P_1, P_2, P_3 are gross profits (before deduction of depreciation) in the respective departments, W_1, W_2, W_3 — the respective wages; P and W aggregate profits and wages, and finally I — gross investment, C_k — capi-

talists' consumption, C_w — workers' consumption and Y — gross national income (before deduction of depreciation).

1	2	3	
P_1 W_1	P_2 W_2	P_3 W_3	P W
I	C_k	C_w	Y

II

It will be assumed, as Marx does, that the workers do not save. Moreover, we shall disregard the problem of possible piling up of stocks of unsold goods as only a passing phenomenon. It is then easy to arrive at the fundamental Marxian " equation of exchange " between Departments 1 and 2 on the one hand and Department 3 on the other.

Profits in the latter, P_3, are materialised in the wage goods which are left to the capitalists of that department after payment of wages W_3 which absorb an equal amount of wage goods. Thus the wage-goods of the value P_3 are sold to the workers of Departments 1 and 2, that is :

$$(1) \qquad\qquad P_3 = W_1 + W_2$$

Marx considers this equation in the context of expanded reproduction proceeding at a given constant rate r. It is easy to see, however, that the equation holds good under *all* circumstances as long as there is no piling up of stocks of unsold goods, as mentioned above.

Considered in this general context equation (1) leads to a proposition that — given the distribution of income between profits and wages in the three departments — investment I and consumption of capitalists C_k determine profits and the national income. Indeed, let us add $P_1 + P_2$ to both sides of equation (1). We obtain :

$$P_1 + P_2 + P_3 = P_1 + W_1 + P_2 + W_2$$

Hence :

$$(2) \qquad\qquad P = I + C_k$$

Moreover, if we denote $\dfrac{W_1}{I}, \dfrac{W_2}{C_k}, \dfrac{W_3}{C_w}$ by w_1, w_2, w_3 respectively, we obtain from equation (1) :

$$(1 - w_3) C_w = w_1 I + w_2 C_k$$

Consequently, we have for the consumption of wage goods :

$$(3) \qquad\qquad C_w = \frac{w_1 I + w_2 C_k}{1 - w_3}$$

and for the national income :

$$(4) \qquad Y = I + C_k + C_w = I + C_k + \frac{w_1\,I + w_2\,C_k}{1 - w_3}$$

Thus the national income (or product) Y which can be sold and the profits P which can be realised are determined in all circumstances (and not only in a state of uniformly expanding reproduction) by the level of investment I and consumption of capitalists C_k (given the distribution of income between wages and profits). A question may be raised as to why equations (2) and (4) must be interpreted in this way and not the other way around, *i.e.*, that investment and consumption of capitalists are determined by profits and national income. The answer to this rather crucial query is as follows.

Investment and capitalists' consumption in the short period consi-dered are the outcome of decisions taken in the past and thus should be considered as given. With regard to investment, this follows directly from the time-lag dependent on the period of construction. But changes in capitalists' consumption also follow those in profits with some delay. Now, sales and profits in a given period cannot be a direct outcome of past decisions : the capitalists can decide how much they will invest and consume next year but they cannot decide how much they shall sell and profit. The independent variables in a given period are investment and capitalists' consumption. It is these magnitudes that through the equations (2) and (4) determine the levels of national income and profits which can be rea-lised.

III

The decisions of capitalists with regard to their investment and consump-tion are made in '' real '' rather than in money terms, that is I and C_k should be calculated in stable prices. If w_1, w_2, w_3 are constant and money wage rates in all three departments change in the same proportion, the same is true in this case of prices of the produce of these departments. More-over, as is easy to see, equations (2) and (3) will hold also in '' real '' terms. Any increase in '' real '' investment or capitalists' consumption results under these circumstances, in an increase in output of Department 3, C_w to provide for a surplus of this department P_3 sufficient to meet the demand generated by the higher wage bills in Departments 1 and 2, *i.e.*, $W_2 + W_3$.

However, such repercussions of an increase in I or C_k are obviously possible only if there exist unused capacities in Department 3. Imagine that such is not the case. Then C_w is fixed in real terms, *i.e.*, is equal to a constant B. In this case the increase in money value of $W_1 + W_2$ will cause a rise in prices rather than in production of wage goods. The result will be that the '' real '' value of W_1, W_2 and W_3 will be reduced as compared with the levels which would be achieved if unused capacities existed in

Department 3.   Consequently $w_1 = \dfrac{W_1}{I}$, $w_2 = \dfrac{W_2}{C_k}$, $w_3 = \dfrac{W_3}{C_w} = \dfrac{W_3}{B}$, where all magnitudes involved are to be now interpreted in " real " terms, will decline in the proportion reciprocal to the increase in the prices of wage goods.   Equation (3) can now be written in the form :

$$\frac{w_1\, I + w_2\, C_k}{I - w_3} = B$$

When I and/or C_k decrease, w_1, w_2 and w_3 decline in such a proportion as to render the left hand side of the equation equal to B [1].

Sections II and III represent in fact the gist of the modern theory of effective demand.   As will be seen, this theory may be derived in full from the Marxian equation (1) representing the exchange between Departments 1 and 2 on the one hand and Department 3 on the other, if this equation is considered in the general context rather than in that of uniformly expanding reproduction.

IV

Let us now turn to the significance of the equations (2) and (4) just in the latter context, *i.e.*, in the process of a uniform accumulation of capital.   Let us denote the " real " stock of capital by K, the rate of net accumulation by r and the rate of depreciation by δ.   In this case we may write the " equation of accumulation ", recalling that I is investment *gross* of depreciation, in the form :

$$(5) \qquad\qquad I = (r + \delta)\, K$$

Since we are considering the long-run process of growth, let us postulate that capitalists' consumption C_k is proportional to profits P.   Since according to formula (2) the latter are equal to $I + C_k$ it follows that C_k bears a constant relationship to I.   We thus have :

$$C_k = m\, I$$

In consequence we may write equation (4) in the form :

$$(6) \qquad Y = (1 + m)\, I + \frac{I\,(w_1 + m\, w_2)}{1 - w_3} = I \left(1 + m + \frac{w_1 + m\, w_2}{1 - w_3} \right)$$

and substituting in it for I its value from equation (5) we obtain :

$$(7) \qquad\qquad Y = K\,(r + \delta) \left(1 + m + \frac{w_1 + m w_2}{1 - w_3} \right)$$

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1. In a socialist economy prices of consumer goods are *always* fixed relative to wages in such a way as to secure a full utilisation of the productive capacity B, *i.e.*, the equation $\dfrac{w_1\, I}{1 - w_3} = B$ is permanently fulfilled (C_k obviously equals 0 in this case).

The national income Y thus bears a constant relationship to the stock of capital K (provided that w_1, w_2, w_3 do not change [2]). With a given relationship of productive capacity to the stock of capital the degree of utilisation of equipment is constant. Thus if capital equipment is satisfactorily utilised in the initial position, this state of affairs is maintained in the course of expanded reproduction and the problem of effective demand does not arise.

It is this approach that is inherent in many contemporary theories of economic growth. In particular if we differentiate the equation (7) we obtain :

$$\frac{d\,Y}{d\,K} = \frac{Y}{K} = r\frac{Y}{r\,K}$$

Now, with a constant satisfactory utilisation of equipment, $\frac{d\,K}{d\,Y}$ is the so-called capital-output ratio which we denote by R. Moreover, r K is the *net* investment and thus $\frac{r\,K}{Y}$ is the relative share of accumulation in the national income which we shall denote by a. We thus have :

$$\frac{1}{R} = \frac{r}{a}$$

or

$$r = \frac{a}{R}$$

which is the basic formula of the Harrod-Domar theory (in which, however, the coefficient a represents the " propensity to save of the population " rather than the ratio of net accumulation out of profits to the national income which depends on its distribution between capitalists and workers).

In fact many of the contemporary theories of growth are simply variations on the theme of Marxian schemes of expanded reproduction which are represented in this paper by equations :

(1) $\qquad\qquad\qquad W_1 + W_2 = P_3$

and

(5) $\qquad\qquad\qquad I = (r + \delta)\,K$

V

The repercussions of changes in investment and capitalists' consumption described in section II do not raise, I believe, any major misgivings. In contrast to this, the moving equilibrium described in section III depends on the very far-reaching assumption that capitalists are willing to engage

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2. If the productive capacities of all three departments expand at the same rater the shortage of wage-goods discussed in the preceding section will not come into the picture.

in investment which increases their capital at a constant rate r per annum. What happens, however, if having become more cautious (perhaps under the influence of a change in the social structure of their class) they decide to reduce investment from $(r + \delta)\, K$ to $(r' + \delta)\, K$ where $r' < r$?

It follows directly from formula (7) that $\dfrac{Y}{K}$ and thus the degree of utilisation of equipment declines in the proportion $\dfrac{r' + \delta}{r + \delta}$ as a result of the decline of effective demand. It is clear that in this situation the " cautious " capitalists will not be any more agreeable to a lower rate of accumulation r' but will reduce it further to $r'' < r'$, and this will in turn affect correspondingly the degree of utilisation of equipment.

Some economists tend to consider this phenomenon as a down-swing phase of the business cycle which takes place around the initial path of growth. However, such a proposition is not well founded : there is no reason why having left the initial unstable path, investment must fluctuate around it rather than around the depreciation level δK. Or to put it in Marxian terms : why cannot a capitalist system, once it has deviated downwards from the path of expanded reproduction, find itself in a position of a long-run simple reproduction?

In fact we are absolutely in the dark concerning what will actually happen in such a situation as long as we have not solved the problem of determinants of investment decisions. Marx did not develop such a theory but nor has this been accomplished in modern economics. Some attempts have been made in the development of the theory of cyclical fluctuations. However, the problems of the determination of investment decisions involving the elements associated with the long-run trend are much more difficult than in the case of the " pure business cycle " (*i.e.*, in a system which in the long run is subject to simple reproduction). I myself tried to do something along these lines but I consider my work in this field to be definitely of a pioneer nature [3]. One thing, however, is clear to me : the long-run growth of the national income involving satisfactory utilisation of equipment in a capitalist economy is far from obvious.

VI

That Marx was deeply conscious of the impact of effective demand upon the dynamics of the capitalist system follows clearly from this passage of the third volume of the *Capital* : " The conditions of direct exploitation and those of the realisation of surplus-value are not identical. They are separated not only by time and space but logically as well. The former

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3. A new paper of mine on the subject appeared in the June issue of the *Economic journal*.

are limited merely by the productive capacity of the society, the latter by the proportions of various branches of production and by consumer power of the society ''.

However, he did not systematically scrutinise the process described by his reproduction schemes from the point of view of the contradictions inherent in capitalism as a result of the problem of effective demand.

It is one of his most prominent followers, Rosa Luxemburg, who expressed very definite and even extreme views on the subject : she rejected altogether the possibility of long-run expanded reproduction if no `` external markets '' are in existence. By `` external markets '' she understood those outside the world capitalist system consisting not only of underdeveloped countries but also of the non-capitalist sectors of developed capitalist economies, for instance, peasant agriculture as well as government purchases.

Her argument suffers from the fact that she considers investment decisions as being made by the capitalist class as a whole and this class is frustrated by the knowledge that finally there is no market for the economic surplus.   However, her scepticism as to the possibility of long-run expanded reproduction is valuable because the self-propelled growth of capitalist economy cannot, indeed, be taken for granted.   If this economy expands at all without the assistance of `` external markets '', this, to my mind, is due to certain aspects of technical progress which, however, do not necessarily assure a satisfactory long-run utilisation of equipment.

Nor should the significance of `` external markets '' in the development of capitalism be disregarded.   In particular, in present-day capitalism the `` external markets '' in the form of government expenditure, especially on armaments, play an important role in the functioning of capitalist economies.   This expenditure to the extent that it is financed by loans, or even by taxation of capitalists, contributes to the solution of the problem of effective demand because its effect is not offset by the decline in investment and consumption.   (The latter would be the case if this expenditure were financed by indirect or direct taxation of workers.)   Thus today the `` external markets '' in this particular form are even of greater significance for expanded reproduction than at the time when Rosa Luxemburg propounded her theory.

The high degree of utilisation of resources resulting in fact from these government-made `` external markets '' has a paradoxical impact upon Western economic theory.   It creates an atmosphere favourable to the construction of models for the growth of `` laissez faire '' capitalist economies unperturbed by the long-run problem of effective demand.

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